Magnetoresistive waves in plasmas

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The self-generated magnetic field of a current diffusing into a plasma between conductors can magnetically insulate the plasma. Propagation of magnetoresistive waves in plasmas is analyzed. Applications to plasma opening switches are discussed.

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Penetration of a current and its associated magnetic field into a plasma is diffusive.[1] If the current through the plasma is sufficiently high, the magnetic field generated by the current can magnetically trap electrons and even ions. When this happens, the plasma resistivity increases, and the plasma is said to be magnetically insulated. Magnetic insulation is an often observed and essential feature of such applications as high-current diodes and spark-gap plasmas. This letter analyzes the propagation of magnetoresistive, or magnetic insulating waves into plasmas including self-field effects. Previous work on magnetic insulation primarily addressed equilibria[2,3] rather than waves. An obvious application of a magnetoresistive wave is a plasma opening switch, which is addressed by this analysis.

For the application of a plasma opening switch, we are interested in magnetic fields intense enough to magnetize the plasma and dominate plasma inertia effects. Assuming constant plasma density in the region of intense magnetic field is reasonable until the pressure of the plasma swept ahead of the field becomes comparable to the magnetic pressure.

Besides neglecting plasma dynamics, we also neglect nonuniform plasma heating. Consequently, the temperature dependence of the plasma conductivity is not manifested. Relativistic effects and displacement currents are taken to be negligible. Instabilities, such as Rayleigh–Taylor and tearing, are not considered here.

The model is one dimensional with spatial variation assumed in the direction of field penetration only. In the model, a uniform plasma fills the half-space $x \geqslant 0$ between conducting planes at $y = 0$ and $y = d$ as shown in Fig. 1(a). Plasma current flows in the y direction, driven externally by an electric field $E\hat{y}$ between the conductors. The current generates a magnetic field $B\hat{z}$. The quasi-one-dimensional approximation[4] depends upon the following conditions being satisfied: The channel length (x direction) is much greater than the height and width; the channel cross-sectional area does not vary quickly; transverse pressure differences are small; the magnetic Reynold's number is much less than 1. In this approximation, Maxwell's equations become

$$\frac{\partial B}{\partial x} = -\frac{4\pi J}{c}, \tag{1}$$

$$\frac{\partial E}{\partial x} = -\frac{1}{c}\frac{\partial B}{\partial t}, \tag{2}$$

where $J\hat{y}$ is the current density.

The plasma is described by nonrelativistic collisional transport theory. If the Hall field[5] is shorted out by the conducting surfaces, then Ohm's law for the collisional, quasi-neutral, weakly ionized plasma may be written

$$J = \sigma E, \tag{3}$$

where the conductivity of the electrons and ions in the magnetic field is given by[5,6]

$$\sigma = ne^2\left(\frac{\tau_e/m}{1+\omega_{ce}^2\tau_e^2} + \frac{\tau_i/M}{1+\omega_{ci}^2\tau_i^2}\right). \tag{4}$$

Here n and e are the electron (and ion) density and charge. The mass, collision time, and gyrofrequency of electrons are m, τ_e, and $\omega_{ce} = eB/mc$, and of ions are M, τ_i, and $\omega_{ci} = eB/Mc$. Because collision times are typically ordered as $\tau_e/\tau_i \approx (m/M)^{1/2}$, electron conductivity dominates as long as $\omega_{ce}\tau_e \lesssim (M/m)^{1/2}$. Ion conductivity dominates in more intense fields. Because the Hall field is shorted out, the conductivity decreases with increasing magnetic field. This effect provides the magnetoresistance of the plasma.

If the conducting electrodes are segmented and driven by separate power supplies, then the Hall field will not be shorted out, and the electron conductivity will be indepen-

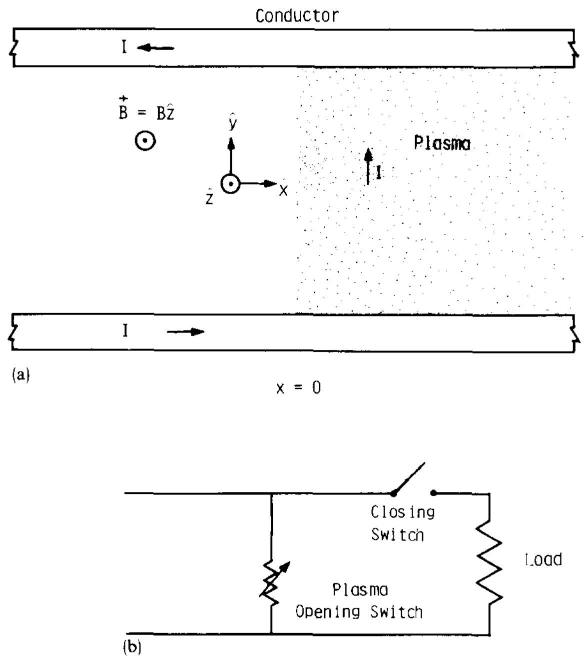

FIG. 1.(a) Geometry of quasi-one-dimensional field penetration model. (b) Equivalent circuit for plasma opening switch.

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dent of magnetic field.[4,5] Where ion conductivity dominates at high field strengths, however, the conductivity is field dependent, and a magnetoresistive wave is driven in qualitatively the same manner. Only the case of a shorted Hall field is treated here.

If a magnetoresistive wave traverses the whole plasma in a channel, then the current can be quenched across the plasma and shunted to a load in parallel. In this manner, the plasma serves as an opening switch, as illustrated schematically in Fig. 1(b).

Combining Eqs. (1)–(3) gives the nonlinear diffusion equation describing propagation of a magnetic wave into a plasma as

$$\frac{\partial}{\partial x}\left(\frac{1}{\sigma}\frac{\partial B}{\partial x}\right)=\frac{4\pi}{c^2}\frac{\partial B}{\partial t}, \tag{5}$$

$$\frac{\partial}{\partial \zeta}\left[\frac{(1+\beta^2)(1+\epsilon^2\beta^2)}{1+\epsilon\beta}\frac{\partial \beta}{\partial \zeta}\right]=\frac{\partial \beta}{\partial \zeta}, \tag{6}$$

where $\beta \equiv \omega_{ce}\tau_e$, $\tau \equiv t/\tau_e$, and the coordinate $\zeta \equiv x/\lambda$ is x normalized to the plasma skin depth $\lambda \equiv (mc^2/4\pi ne^2)^{1/2}$. We have made use of the ordering $\epsilon \equiv m\tau_i/M\tau_e \ll 1$ to simplify the conductivity.

The boundary conditions are found by integrating Eq. (1). At $\zeta=\infty$, we require $\beta=0$. At $\zeta=0$, we find $\beta=\beta_0$, where $\beta_0 \equiv 4\pi e I \tau_e/mc^2$ and $I(t)$ is the current per unit width in the plasma. We solve Eq. (6) for two cases of interest: constant current causing a magnetoresistive wave to diffuse inward and increasing current causing a magnetoresistive wave to propagate inward at constant velocity.

If the current is constant, then Eq. (6) may be written in terms of a new diffusion coordinate $\xi \equiv \zeta/\tau^{1/2}$ as

$$\frac{d}{d\xi}\left[\frac{(1+\beta^2)(1+\epsilon^2\beta^2)}{1+\epsilon\beta^2}\frac{d\beta}{d\xi}\right]=-\frac{\xi}{2}\frac{d\beta}{d\xi}. \tag{7}$$

Some properties of this equation may be deduced by inspection. Because each extremum in β is also a saddle point, β must decrease monotonically with ξ. Therefore, the peak of the magnetic field is always at the plasma boundary. However, because $d^2\beta/d\xi^2<0$ at the boundary, the peak in $-d\beta/d\xi$ appears inside the plasma away from the boundary. Therefore, the current density $J=-(ne\lambda/\tau_e)(\tau_e/t)^{1/2}\times d\beta/d\xi$ peaks inside the plasma at one or more surfaces that diffuse inward with time. The penetration of a current maximum into the plasma is caused by the diffusion of the magnetoresistive wave behind it. This diffusion may be compared with nonlinear magnetic diffusion in metals[6] and with propagation of thermal waves from a heat source.[7]

In the limit $\beta_0 \ll 1$, the conductivity is nearly independent of the magnetic field, and the field diffuses inward approximately as $\beta \approx \beta_0[1-\mathrm{erf}(\xi/2)]$. In this weak field limit, the current density $J=(4\beta_0 ne\lambda/\tau_e)(\tau_e/\pi t)^{1/2}\exp(-\xi^2/4)$ has a maximum at the plasma boundary. The numerical solution of Eq. (7) for the more general case of a field-dependent conductivity is shown in Fig. 2. The spike in current density at the foot of the magnetic wave is caused by electrons crossing in the region where they have not yet become trapped. A secondary peak in current density, which is caused by ions crossing, sometimes occurs at higher magnetic field strengths just beyond the region of ion trapping.

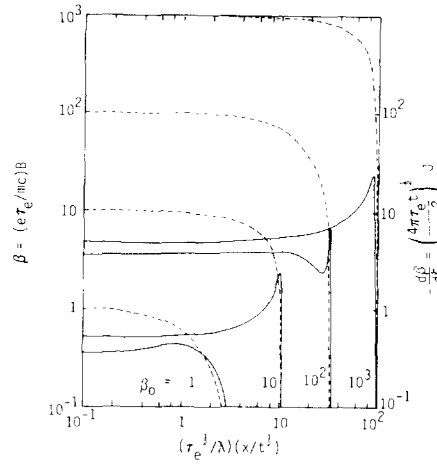

FIG. 2. Profiles of normalized magnetic field β (dashed curves) and normalized current density $-d\beta/d\xi$ (solid curves) in the normalized diffusion coordinate for several indicated values of magnetic field at the plasma boundary and for $\epsilon=0.006$.

If the plasma current increases, then a magnetoresistive wave can propagate into the plasma at constant velocity rather than diffusing in. In that case, Eq. (6) written in terms of a constant dimensionless wave velocity V and a new coordinate $\eta \equiv \zeta - V\tau$ may be integrated immediately to give

$$\frac{d\beta}{d\eta}=\frac{-V(1+\epsilon\beta^2)\beta}{(1+\beta^2)(1+\epsilon^2\beta^2)}, \tag{8}$$

and integrated again to give

$$\eta=\frac{1}{2V}\left[\frac{1}{\epsilon}\ln\left(\frac{1+\epsilon\beta_0^2}{1+\epsilon\beta^2}\right)+\ln\left(\frac{\beta_0}{\beta}\right)^2+\epsilon(\beta_0^2-\beta^2)\right]. \tag{9}$$

Here, $\beta_0=4\pi e I(0)\tau_e/mc^2$ is the value of β at $\eta=0$, and the velocity of the magnetoresistive wave is $v=\lambda V/\tau_e$. The solutions, Eqs. (8) and (9), are shown in Fig. 3.

Again, it is easily seen from Eq. (8) that β decreases monotonically with η. Differentiating Eq. (8) shows that $-d\beta/d\eta$ has local maximum values of $V/2$ where $\beta \approx 1$ and where $\beta \approx 1/\epsilon$. Therefore, the current density has local maximum values of $J_{\max}=nev/2$ at these magnetic field strengths. The maximum at $\beta \approx 1$ is caused by the electrons crossing where they have not yet become trapped; the maximum at $\beta \approx 1/\epsilon$ is caused by the ions crossing where they

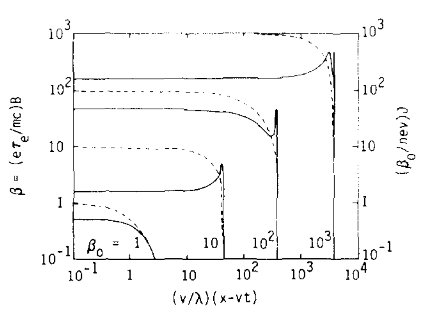

FIG. 3. Profiles of normalized magnetic field (dashed curves) and normalized current density (solid curves) in the normalized traveling-wave coordinate for several indicated values of magnetic field at the plasma boundary and for $\epsilon=0.006$.

have not yet become trapped. Of course, the maximum in current density caused by ions will not be observed unless $\beta_0 > 1/\epsilon$. Nor will the electrons cause a maximum of current density to occur within the plasma unless $\beta_0 > 1$. These observations are all apparent in Fig. 3. In the asymptotic limit of large fields, in which $\epsilon\beta(x=0) \gg 1$ and the conductivity near the boundary is dominated by ions, the current must increase with time as

$$I \sim (nev\lambda)\ (2t/e\tau_e)^{1/2} \tag{10}$$

in order to maintain the propagating wave.

The velocity of the propagating wave depends on the electrical power used to drive it. From Eq. (2), we find

$$E = (v/c)B\ . \tag{11}$$

At the plasma surface, therefore, the potential across the conductors is

$$\phi_0 = (v/c)(4\pi I/c)d\ . \tag{12}$$

Current as a function of time is found by evaluating Eq. (9) at the plasma boundary. In the asymptotic limit of large fields, Eq. (10) gives the constant velocity of the magnetoresistive wave as

$$v \sim (2\pi\epsilon\tau_e/nmc^2)^{1/2}\ [I(t)/t^{1/2}]\ . \tag{13}$$

In the same asymptotic limit, the linear power density in the plasma is

$$P = I\phi_0 = \frac{4\pi d}{c^2}\left(\frac{2\pi\epsilon\tau_e}{nmc^2}\right)^{1/2}\frac{I^3(t)}{t^{1/2}} = \left(\frac{2nmd}{\epsilon\tau_e}\right)v^3 t. \tag{14}$$

As an example, suppose that the linear current density is increasing as $I = (1\ \text{MA/cm})(t/1\ \mu\text{s})^{1/2}$ in a 1-eV aluminum plasma at 10^{16} cm^{-3}. Then a magnetoresistive wave will penetrate the plasma at a constant speed of about 20 cm/μs at linear current densities above about 1 MA/cm. The resistance of the plasma does not increase rapidly until the magnetoresistive wave reaches the end of the plasma. If the plasma is used as an opening switch, the switching time scales as the scale length of the front of the magnetoresistive wave divided by the velocity of the wave.

In conclusion, we have analyzed propagation of magnetoresistive waves into plasmas in which Hall fields are shorted out, including self-field effects and ion as well as electron conductivity. Two analytic solutions were presented in the quasi-one-dimensional approximation. One describes a constant current diffusing into a plasma. The other describes an increasing current propagating into a plasma at constant velocity. Magnetoresistive waves may be capable of quickly opening high-current circuits at high voltage.

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