Accelerating particles with transverse cavity modes

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In optical cavity modes the fields in pulsed operation are limited by skin current heating to large values on order of 0.1–1 GV/m. Moreover, efficient transfer of energy from such modes to relativistic particles seems possible. A single cavity electron accelerator based on such modes would show an appreciable energy gain.

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This paper presents an interesting method for accelerating relativistic particles to ultrarelativistic energies. The acceleration mechanism is based on the large electric field strength that is obtainable in optical cavities filled with transverse modes, i.e., light bouncing back and forth between mirrors. The corresponding mode in microwave cavities is the transverse magnetic mode. The maximum electric field depends somewhat on pulse length and frequency; estimates[1] from skin current heating yield a peak value of at least 5×10^8 V/m at 100 MHz in microsecond pulse operation. The largest electric fields occur away from the wall: at the wall the transverse electric field is a factor δ/λ, smaller due to interference of the incoming with the reflected wave (δ is the skin depth, λ is the wavelength).

For these large fields the electromagnetic energy in a cavity of reasonable size is tens of kilojoules; filling the cavity on a microsecond timescale demands an extremely powerful rf source. In the following it will be assumed that the maximum allowable field strength can be reached, even though this assumption is unrealistic at present.

For a powerful accelerator a large field is not enough; there must also be a net energy transfer to a charged particle passing through. The higher transverse modes familiar from laser resonance cavities seem particularly suitable for energy transfer as shown below.

Higher optical modes manifest themselves in the focal plane by a spatial structure with a number of interesting lobes proportional to the mode number (e.g., Figs. 6 and 7 in Ref. 2). With a suitable initial phase, the particle is accelerated in the first lobe; in a later lobe the particle might decelerate only to be accelerated again in another lobe further down the line. However, the net effect of the accelerations and decelerations is an energy gain comparable to $eE_p\lambda$, where E_p is the peak value of the electric field, at the center of the cavity λ is the wavelength, and e is the electronic charge.

In the sequel we discuss in some detail these main ideas behind the transverse accelerator, namely, the higher radiation modes in a suitable resonant cavity, and the energy transfer to relativistic particles.

Figure 1(a) shows a suitable cavity; it looks like a cylindrically symmetric keg or barrel. The electric field vector points in the axial z direction, and oscillates up and down with the resonant frequency. The particle is assumed to move along the axis at the light speed.

The cavity can be conceptually generated from two near confocal mirrors, shown in Fig. 1(b), by rotation about the z axis. Confocal mirrors are a favored resonator in laser applications, and their properties are well known.[2–5] In the optical regime the wavelength is much less than typical mirror dimensions, and in particular less than the distance between the mirrors. In this situation various almost lossless transverse modes can be built up between the mirrors, even though the cavity is open and energy could escape; the lossless modes correspond to beams of light bouncing back and forth between the mirrors. Their main energy loss comes from diffraction, which is small when the light intensity does not reach the mirror edges. Typically, the diffraction loss corresponds to a quality factor $Q\simeq 2000$. The energy from modes corresponding to light not hitting the mirrors is lost in essentially one transit time.

The accelerator cavity, however, should contain microwave energy with a wavelength on the order of 0.1–1 m. Also, the accelerator cavity cannot be too large, and therefore the wavelength is smaller than cavity dimensions by a

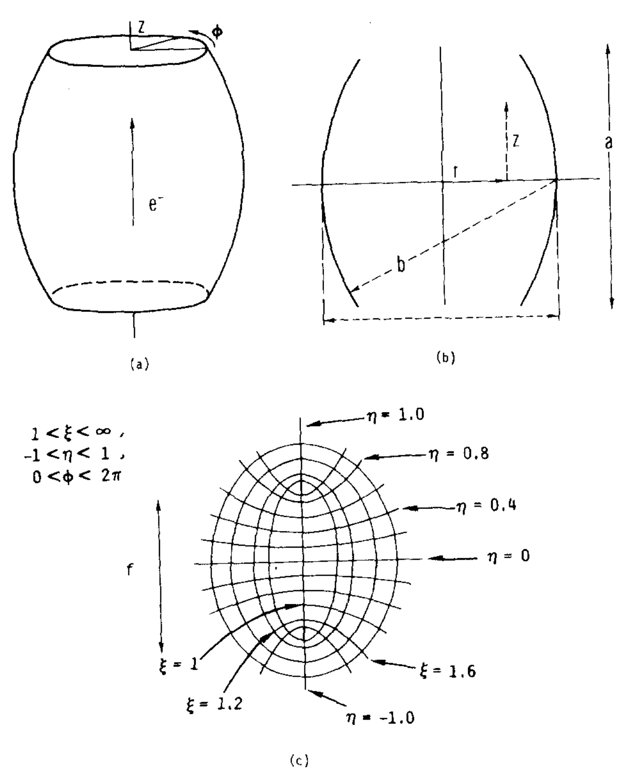

FIG. 1. (a) Accelerator cavity with relativistic electron. (b) Confocal mirrors, or cross-cut through cavity. (c) Prolate spheroidal coordinates.

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factor of 10 at most. Various approximations that are useful for laser cavities can no longer be used for the accelerator cavity. Instead, the rf cavity can be analyzed in prolate spheroidal coordinates[5–9] shown in Fig. 1(c). From Figs. 1(a)–1(c) it is clear that the proposed cavity can be described by one of the ellipsoids obtained by keeping the coordinate ξ fixed at an appropriate value ξ_w: for confocal mirrors $\xi_w = \sqrt{2}$. (Note that the spheroidal coordinate ξ increases from unity to infinity in the radial direction, while the coordinate η, $-1 \leqslant \eta \leqslant 1$, measures in the axial direction.)

The spheroidal wave functions, for wavelengths short compared to cavity dimensions, lead[7–9] along the axis to the well-known field structure,

$$E_z(z,t) = E_p N_n H_n(\zeta)\exp(-\tfrac{1}{2}\zeta^2)\cos\omega_n t, \tag{1}$$

where $\zeta = 2z/W$ is the coordinate normalized by the waist size $W = (f\lambda/\pi)^{1/2} = (\lambda/\pi)\gamma^{1/2}$; here f is the focal length and $\gamma = f\omega_n/2c$ is an auxiliary constant containing the resonant frequency ω_n which depends on the mode number n. The functions $H_n(\zeta)\exp - \frac{1}{2}\zeta^2$ are the Hermite polynomials times the Gaussian familiar from the harmonic oscillator in quantum mechanics, and N_n is a normalization constant such that E_p is the peak electric field.

Given the field structure along the axis, it is straightforward to calculate the energy transfer to a relativistic particle. The particle always travels with the light speed c, $z = ct$, and therefore, setting $\omega_n t = s$,

$$\Delta E = c\int \frac{dp}{dt}\,dt = \frac{ceE_pN_n\gamma^{1/2}}{\omega_n} \times \left|\int_{-\infty}^{\infty} ds\, H_n(s)\exp(-\tfrac{1}{2}s^2 + i\gamma^{1/2}s)\right|. \tag{2}$$

The momentum transfer is the absolute value of the Fourier transform over the spatial electric field. The Fourier transform is[10]

$$\left|\int ds\, H_n(s)\exp(-\tfrac{1}{2}s^2 + i\gamma^{1/2}s)\right| = (2\pi)^{1/2}H_n(\gamma^{1/2})\exp(-\tfrac{1}{2}\gamma), \tag{3}$$

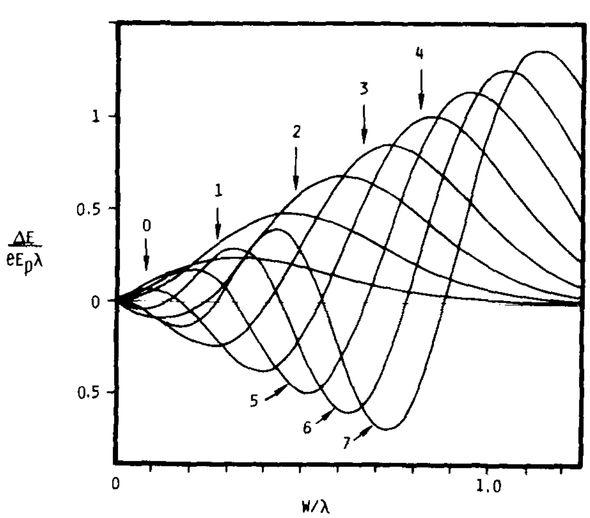

FIG. 2. Energy transfer to a relativistic electron normalized to E_p as function of waist size W in units of wavelength for axial modes. $n = 0 - 7$.

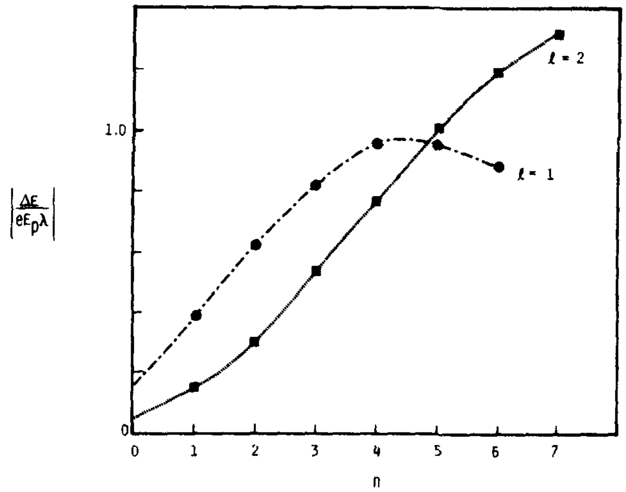

FIG. 3. Energy transfer to a relativistic electron normalized to $eE_p\lambda$ as function of axial mode number n for radial mode numbers $\lambda = 1,2$ in a confocal cavity.

and the energy increase becomes, in terms of the waist size in units of wavelength,

$$\Delta E = eE_p\lambda_n \frac{N_n}{(2\pi)^{1/2}} \frac{\pi W}{\lambda} H_n\left(\frac{\pi W}{\lambda}\right) \times \exp - \frac{\pi^2}{2}\left(\frac{W}{\lambda}\right)^2. \tag{4}$$

The numerical factor multiplying $(eE_p\lambda_n)$ is given in Fig. 2. The factor is of order unity for waist sizes comparable to half the wavelength. Larger modes show particularly good coupling, even exceeding unity: this should be compared to 0.5 if the spatial structure were a single square wave of length $\frac{1}{2}\lambda$, or to $\pi^{-1} \simeq 0.32$ for one lobe of a sinusoid. Passing through the higher modes the particle picks up energy from all the lobes, but it is decelerated only slightly.

The waist size is not a free parameter; it is given by the cavity resonances through $W/\lambda = \gamma^{1/2}/\pi$. The parameter γ is determined by the boundary conditions in the radial direction, and by the mode number n. A rough approximation,[11] valid for large l in a confocal resonator, is

$$\gamma \simeq j_{0l} + \tfrac{1}{8}(2n+1)\pi, \tag{5}$$

where j_{0l} is the lth root of the zero-order Bessel function $J_0(j_{0l}) = 0$. A corresponding estimate for small l does not seem to be available: using Eq. (5) for $l = 1$ and $l = 2$, the calculated energy transfer is given in Fig. 3: the maximum coupling 1.2 is as large as the largest possible value from Fig. 2, and about four times larger than the coupling from a single lobe. The coupling efficiency depends also on the cavity shape, and the accuracy of the approximation.[5] Therefore the values in Fig. 3 will change somewhat in a more accurate calculation.

The cavity's resonant frequency determines the wavelength $\lambda_n = 2\pi c/\omega_n$, which implies an additional dependence on the mode number through the factor $(eE_p\lambda_n)$. In any case, the wavelength is somewhat under control because it is proportional to cavity dimensions; these can be chosen arbitrarily, subject to extraneous constraints such as size or cost.

The conclusion from Eq. (4) is that transverse cavity modes can accelerate relativistic particles in a fairly efficient fashion. The energy gain per pass through the accelerator is then of order $(eE_p\lambda_n)$, with a coupling factor or order unity.

Further feasibility studies are underway to support the present calculations. If the attainable field strength and coupling factors are indeed close to those calculated here, a very interesting accelerator cavity would result, with an accelerating gradient of order 150 MeV, in a meter long cavity.

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