Universal formula for quasi-static density perturbation by a magnetoplasma wave

A. N. Kaufman, J. R. Cary,[a] and N. R. Pereira[b]

Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
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The general expression for the ponderomotive Hamiltonian is used to obtain the quasi-static quasi-neutral density change caused by the ponderomotive force of a cold magnetoplasma wave of arbitrary frequency and polarization: $\delta n(\mathbf{x}) = -[|\mathbf{E}(\mathbf{x})|^2 - |\mathbf{B}(\mathbf{x})|^2]/4\pi(T_e + T_i)$.

In studying the modulation of a finite-amplitude plasma wave, a number of authors have calculated the quasi-static quasi-neutral second-order density perturbation $\delta n(\mathbf{x})$ produced by the ponderomotive force of the modulation. With the representation for the potential

$$\phi(\mathbf{x}, t) = \phi(\mathbf{x}) \exp(-i\omega t) + \text{c.c.} \tag{1}$$

of a longitudinal magnetoplasma wave, the result

$$\delta n(\mathbf{x}) = -\frac{|\nabla\phi(\mathbf{x})|^2}{4\pi(T_e + T_i)} \tag{2}$$

has been obtained by Morales and Lee[1] for lower-hybrid waves, and by Shukla[2] for electron magnetoplasma waves. The former authors remarked on the identity of formula (2) with the familiar expression for Langmuir wave modulation in unmagnetized plasma.

It is natural to inquire into the universality of formula (2). In this Note, we show that it does indeed apply to any longitudinal cold-plasma wave (for a single ion species); i.e., the three solutions $\omega(\theta)$ of $\epsilon_l(\omega, \theta) = 0$, where $\epsilon_l \equiv \hat{k} \cdot \boldsymbol{\epsilon}(\omega) \cdot \hat{k}$.

More importantly, we show that formula (2) can simply be generalized to apply to a cold-plasma wave of any polarization, i.e., to a wave with nonzero $\nabla \times \mathbf{E}$. Here, we use a local plane-wave representation

$$\mathbf{E}(\mathbf{x}, t) = \tilde{\mathbf{E}}(\mathbf{x}) \exp(i\theta(\mathbf{x}) - i\omega t) + \text{c.c.} \tag{3a}$$

with

$$\mathbf{k}(\mathbf{x}) \equiv \nabla\Theta(\mathbf{x}), \tag{3b}$$

and

$$\tilde{\mathbf{B}}(\mathbf{x}) = (c/\omega)\mathbf{k} \times \tilde{\mathbf{E}}. \tag{3c}$$

The generalization, to be derived, is

$$\delta n(\mathbf{x}) = -\frac{|\tilde{\mathbf{E}}(\mathbf{x})|^2 - |\tilde{\mathbf{B}}(\mathbf{x})|^2}{4\pi(T_e + T_i)}. \tag{4}$$

We note first that it reduces to (2) when $\tilde{\mathbf{B}} = 0$. Secondly, for the transverse unmagnetized case, where

$$|\tilde{\mathbf{B}}|^2 = (kc/\omega)^2 |\tilde{\mathbf{E}}|^2 = (1 - \omega_p^2/\omega^2)|\tilde{\mathbf{E}}|^2,$$

formula (4) becomes

$$\delta n/n = -(e^2/m\omega^2)|\tilde{\mathbf{E}}|^2/(T_e + T_i),$$

the familiar result.[3]

Formula (4) can be used for any cold-magnetoplasma wave, e.g., lower hybrid in the electromagnetic region,[4] fast-magnetosonic whistler,[5] Alfvén,[6] ordinary and extraordinary, etc., so long as (3c) is a valid approximation. [When it is not, use formula (10).]

Our derivation begins with the standard expression for the quasi-static density perturbation, of species s, caused by the ponderomotive potential energy $\Psi_s(\mathbf{x})$ of an oscillation center[7] and by the self-consistent electric potential $\Phi(\mathbf{x})$

$$\delta n_s(\mathbf{x})/n_s^0 = -[\Psi_s(\mathbf{x}) + e_s\Phi(\mathbf{x})]/T_s. \tag{5}$$

For two species (electrons and singly-charged ions), we impose quasi-neutrality ($\delta n_e = \delta n_i, n_e^0 = n_i^0$) to eliminate Φ, and obtain the relation

$$\delta n(\mathbf{x})/n^0 = -\frac{[\Psi_e(\mathbf{x}) + \Psi_i(\mathbf{x})]}{(T_e + T_i)}. \tag{6}$$

Our expression for $\Psi_s(\mathbf{x})$ is based on a useful relation[8] for the ponderomotive Hamiltonian[9] of an oscillation center. In the cold-species limit, Eq. (3) of Ref. 8 reduces to

$$n_s(\mathbf{x})\Psi_s(\mathbf{x}) = -(4\pi)^{-1}\mathbf{E}^*(\mathbf{x}) \cdot \boldsymbol{\chi}_\omega^s(\mathbf{x}) \cdot \mathbf{E}(\mathbf{x}), \tag{7}$$

with the representation $\mathbf{E}(\mathbf{x}, t) \equiv \mathbf{E}(\mathbf{x}) \exp(-i\omega t) + \text{c.c.}$, where $\boldsymbol{\chi}_\omega$ is the well-known[10] cold-species susceptibility. (We note that χ is proportional to density, so that Ψ is density-independent; but the dependence of χ on possibly nonuniform magnetostatic field $\mathbf{B}_0(\mathbf{x})$ appears in Ψ).

Inserting (7) into (6), we have

$$\delta n(\mathbf{x}) = \frac{\mathbf{E}^*(\mathbf{x}) \cdot (\boldsymbol{\chi}_\omega^e + \boldsymbol{\chi}_\omega^i) \cdot \mathbf{E}(\mathbf{x})}{4\pi(T_e + T_i)}. \tag{8}$$

Now, we use the field equation

$$(\boldsymbol{\chi}_\omega^e + \boldsymbol{\chi}_\omega^i) \cdot \mathbf{E}(\mathbf{x}) = -\mathbf{E}(\mathbf{x}) + (ic/\omega)\nabla \times \mathbf{B}(\mathbf{x}), \tag{9}$$

where $\mathbf{B}(\mathbf{x}) = (c/i\omega)\nabla \times \mathbf{E}(\mathbf{x})$, to obtain

$$\delta n(\mathbf{x}) = -\frac{|\mathbf{E}(\mathbf{x})|^2 - |\mathbf{B}(\mathbf{x})|^2 - (c/\omega)\,\mathrm{Im}\nabla \cdot \mathbf{E}^*(\mathbf{x}) \times \mathbf{B}(\mathbf{x})}{4\pi(T_e + T_i)}. \tag{10}$$

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[a] Present address: Los Alamos Scientific Laboratory, Los Alamos, N. M. 87545.

[b] Present address: Dynamics Technology, Inc., Torrance, Calif. 90503.

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Finally, for a local plane wave, with $\mathbf{E}(x) \equiv \tilde{\mathbf{E}}(x) \exp i\Theta(x)$, (3b) and (3c), one may drop the comlex Poynting term in (10), as higher order in $k\nabla \ln \tilde{E}$; the result is then Eq. (4).

Several points should be kept in mind in applying (4): second-order magnetic perturbations may be of significance[11]; the quasi-static assumption may be invalid;[12] the isothermal assumption (5) may be invalid, e.g., for a nonlocalized cavity mode, the temperature is not an adiabatic invariant; for more than one ion species, formulae (2) and (4) generalize to less beautiful forms.

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Effect of ionic temperature on the modulational instability of ion acoustic waves in a collisionless plasma

I. R. Durrani, G. Murtaza, and H. U. Rahman

Department of Physics, Quaid-i-Azam University, Islamabad, Pakistan

I. A. Azhar

Pakistan Institute of Nuclear Science and Technology, Rawalpindi, Pakistan
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Using the Krylov–Bogoliubov–Mitropolski method, the modulational instability of ion-acoustic waves in a collisionless plasma consisting of isothermal electrons and adiabatic ions is studied. It is found that with the inclusion of ion temperature the modulational instability sets in much earlier.

The modulational instability of the ion wave has been observed experimentally.[1] Theoretically, the linear stability criterion of the nonlinear Schrödinger equation shows that the modulational instability sets in only when $k/k_D > 1.47^2$ $(\omega/\omega_{pi} > 0.83)$ assuming the ion temperature to be negligible. For waves with such a large wavenumber, however, the ion Landau damping becomes strong and overwhelms the nonlinear effect. On the other hand, in the experiment[1] the electron temperature was 1.5–2.0 eV the ion temperature was $T_e/10 - T_e/12$, and the modulation was observed around the frequency $\omega/\omega_{pi} = 0.5$. The experimental result therefore cannot be explained by the nonlinear Schrödinger equation derived for a mixture of cold ion fluid and hot isothermal electrons.

In the earlier experiments on ion acoustic solitary waves,[3] it was found that the formation of a soliton was sensitive to the value of the temperature ratio. It was observed that when T_e/T_i is low, say 10, the initial perturbation does not break into solitons but induces turbulent noise. Some theoretical estimates[4] were also made to examine the effect of ionic temperature on the characteristics (amplitude, width, and velocity) of the solitary wave.

In the present note we attempt to determine how the modulational instability is affected by the ion temperature and whether the present experimental result can be explained in this way without having to incorporate Landau damping.

We use the Krylov–Bogoliubov–Mitropolski method as improvised by Kakutani and Sugimoto[2] for a collisionless plasma consisting of adiabatic ions and isothermal electrons.

Such a system is governed by the following dimensionless set of equations:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\sigma}{n}\frac{\partial p}{\partial x} = E,$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + 3p\frac{\partial u}{\partial x} = 0, \quad \frac{\partial E}{\partial x} = n - n_e, \quad \frac{\partial n_e}{\partial x} = -n_e E, \tag{1}$$

where σ is the ratio of the ion temperature T_i to the electron temperature T_e; also n, n_e, u, and E denote, respectively, ion density, electron density, ion fluid velocity, and the electric field which are all normalized

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