A simple derivation of the Pease–Braginskiĭ current

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The Pease–Braginskiĭ current is rederived for a uniform single species Z pinch. The result contains the Alfvén current and the ratio between the collisional and bremsstrahlung cross sections.

The Pease–Braginskiĭ current I_{PB} is a unique value of the current for a fully ionized hydrogen pinch.[1,2] This current corresponds to force equilibrium, expressed by the Bennett relation,[3] and, in addition, the power equilibrium between bremsstrahlung loss and Joule heating. The literature[3,4] typically contains various numerical formulas for I_{PB}, e.g., I_{PB} (MA) $= 0.433\ (\ln \Lambda)^{1/2}$, where $\ln \Lambda$ is the Coulomb logarithm; often $\ln \Lambda$ is implicitly taken as ~ 10 and I_{PB} is given as ~ 1.4 MA.

It should be expected that this unique value of the current is related in some transparent way to nature's current scale, the Alfvén(–Lawson) current I_A. This current scale is defined by the fundamental constants of nature (c, e, μ_0, and the like) as $I_A = ec/r_e$, where the classical radius of the electron $r_e = (e^2/mc^2)[1/4\pi\epsilon_0]$. Here we rederive the Pease–Braginskiĭ current as the natural current scale I_A multiplied by a ratio between the cross sections for multiple collisions ($\sim 8 \ln \Lambda$) and bremsstrahlung ($\sim \alpha F$) [Eq. (6)].

The model pinch consists of a stationary plasma cylinder with uniform temperature T, electron density n_e, ion density $n_i = n_e/Z$, and single charge state Z. Irrespective of any radial gradients the total current I is related to the total line density and temperature by the Bennett relation[5]

$$\mu_0 I^2/4\pi = 2N_e(1 + 1/Z)kT, \tag{1a}$$

where the electron line density $N_e = \pi r_0^2 n_e$ and r_0 is the pinch radius. In terms of the Alfvén current, which can be written as $\mu_0 I_A^2/4\pi = mc^2/r_e$, the Bennett relation becomes

$$\left(\frac{I}{I_A}\right)^2 = 2\left(1+\frac{1}{Z}\right)\left(\frac{kT}{mc^2}\right)(N_e r_e). \tag{1b}$$

The Pease–Braginskiĭ current I_{PB} in this stationary pinch is defined by a balance between resistive (or Joule) heating and bremsstrahlung radiative loss. In the absence of heat conduction, opacity, and the like (as we shall assume), both Joule heating and bremsstrahlung loss are local, collisional processes. It is therefore tempting to express these losses as a local power density of the form $P = n_e n_i v\sigma(\Delta\epsilon)$, where v is some average velocity, σ a typical cross section, and $\Delta\epsilon$ a typical energy transfer per collision.

In the simplest model of Joule heating an electron is accelerated by the electric field until a time $1/\nu_{ei}$, when it suffers a collision and the energy gain is randomized. The power deposition from Joule heating is $P_j = \eta j^2$, where η is the resistivity and j is the current density. Explicitly, $\eta = m\nu_{ei}/n_e e^2$, with $\nu_{ei} = 8 \ln \Lambda (v_0 a_0^2) Z n_e (2\mathscr{R}/kT)^{3/2}$. This useful but nonstandard form is written in atomic units,[6] viz., the Bohr radius $a_0 = \hbar^2/me^2[4\pi\epsilon_0]$, the atomic velocity $v_0 = \alpha c$, where $\alpha = (e^2/c\hbar)[1/4\pi\epsilon_0]$ is the fine structure constant, and the Rydberg energy $2\mathscr{R} = mv_0^2 = \alpha^2 mc^2$.

The Joule power density $P_j = \eta j^2$ is then

$$P_j = n_i n_e v\left[8 \ln \Lambda a_0^2 Z^2 \left(\frac{2\mathscr{R}}{kT}\right)^2\right]\left(\frac{mj^2}{n_e^2 e^2}\right), \tag{2}$$

where the characteristic velocity v is defined by $mv^2 = kT$. The energy loss per collision can be identified with the last term, $\Delta\epsilon_j = mj^2/n_e^2 e^2 = mv_d^2$, where the drift velocity $v_d = j/n_e e$. Then a collision cross section for Joule heating is given by the term in square brackets.

The energy loss per collision can be expressed in terms of the total current I and the electron line density N_e. For a uniform pinch $I = \pi r_0^2 j$ and $N_e = \pi n_e r_0^2$, and $\Delta\epsilon_j = mI^2/N_e^2 e^2$, which is also

$$\Delta\epsilon_j = (mc^2/r_e^2 N_e^2)(I/I_A)^2. \tag{3}$$

The power density for bremsstrahlung can be written in a form reminiscent of Eq. (3),

$$P_b = n_i n_e v a_0^2 Z^2 \alpha F_b (2\mathscr{R})(2\mathscr{R}/mc^2), \tag{4}$$

where the factor $F_b = (2\pi/27)^{1/2}$ in the most accessible derivation,[7] and slightly different for more complete calculations. Although bremsstrahlung is emitted with a continuous spectrum the energy of a typical bremsstrahlung quantum is $\hbar\omega \sim kT$. Therefore kT can be identified with the energy loss per collision $\Delta\epsilon_b = kT$. Then the bremsstrahlung cross section is $\sigma_b = a_0^2 Z^2 \alpha F_b (2\mathscr{R}/kT)^2 (kT/mc^2)$.

Pease–Braginskiĭ equilibrium adds to the Bennett relation the condition $P_b = P_j$, which corresponds to local energy balance in the microscopic processes, viz., $\sigma_b \Delta\epsilon_b = \sigma_j \times \Delta\epsilon_j$. This becomes

$$a_0^2 Z^2 \alpha F_b (2\mathscr{R})^2/mc^2 = a_0^2 Z^2 8 \ln \Lambda \left[\left(\frac{2\mathscr{R}}{kT}\right)^2 \left(\frac{mc^2}{r_e^2 N_e^2}\right)^2 \left(\frac{I}{I_A}\right)^2\right]. \tag{5}$$

The plasma parameters are grouped in the square brackets. The atomic cross section a_0^2 can be divided out, and the plasma parameters disappear on using Eq. (1b). The result is the transparent formula

$$\frac{I_{PB}}{I_A} = \left(\frac{8 \ln \Lambda}{\alpha F_b}\right)^{1/2} 2\left(1 + \frac{1}{Z}\right). \tag{6}$$

The Pease–Braginskiĭ current is the natural current scale I_A, multiplied by the square root of the ratio between the cross sections for the collisions that are responsible for heating, $\sigma_j \propto 8 \ln \Lambda$, and energy loss from bremsstrahlung, $\sigma_b \propto \alpha F_b$.

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Multiple collisions and bremsstrahlung are completely classical processes, but Eq. (6) still contains an essentially quantum mechanical quantity, the fine structure constant $\alpha = (e^2/c\hbar)[1/4\pi\epsilon_0]$. This α comes from a lower limit to the impact parameter in the integration of the bremsstrahlung from an electron passing a fixed charge. This cutoff, which keeps bremsstrahlung finite, was an early triumph of quantum theory. Interestingly enough, the factor $\ln \Lambda$ also results from a cutoff, this time from Debye screening at the upper limit to the impact parameter.

Bremsstrahlung is the only energy loss mechanism in the Pease–Braginskiĭ current I_{PB}. However, bremsstrahlung is usually swamped by line radiation and power balance between line radiation and Joule heating gives rise to a much smaller equilibrium current. Knowing the ratio $K = \sigma_l \Delta\epsilon_l / \sigma_b \Delta\epsilon_b$ between the cross sections and energy losses for line radiation and bremsstrahlung gives an estimate for the Pease–Braginskiĭ current for line radiation, viz., $I^l_{PB} = I_{PB}/K^{1/2}$. This same result is obtained by comparing the power densities.[8] Likewise, I_{PB} increases with the decrease in radiation as a result of opacity,[9] and changes by introducing radial nonuniformities[1,2,10] and anomalous resistivity.[11]

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Experimental study of multiple frequency effects in a free electron laser amplifier

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Measurements are presented of the gain and phase of a free electron laser (FEL) amplifier in the presence of a second, perturbing electromagnetic wave of different frequency. The studies are carried out in the microwave range of 7–11 GHz using a mildly relativistic electron beam of ~ 160 kV energy and a current of ~ 1 A. The presence of the perturbing wave causes serious deterioration of the FEL performance.

Multifrequency studies can shed much light on the nonlinear dynamics and wave–particle interactions taking place in free electron lasers (FEL's). One well known example of such a multifrequency effect is the appearance of sidebands[1–3] in the FEL radiation spectrum. This phenomenon becomes particularly challenging when the radiation propagates in a highly dispersive waveguide (as is the case in our experiments), since now the different frequency waves are associated with different ponderomotive wells, each of which travels at quite a different velocity.[4] FEL amplifiers are usually not beset by sidebands, unlike FEL oscillators in which sidebands are readily excited[1–3] and are of much concern. Thus we can examine multifrequency phenomena in our amplifier in a controlled way by launching two waves from external sources: one corresponding to the resonant FEL frequency f and a second nonresonant perturbing wave of frequency f_0. We then measure changes in the FEL gain at frequency f caused by the presence of the wave at the neighboring frequency f_0. All of our studies reported here were made in the collective (Raman) regime where space charge effects cannot be neglected.

Figure 1 shows a schematic of our experiment.[5] The accelerating potential is supplied by a Marx generator (Physics International Pulserad 615MR), which has a maximum capability of 500 kV and 4 kA. The electron beam is generated by a thermionically emitting, electrostatically focused, Pierce-type electron gun ($\leqslant 250$ kV, $\leqslant 250$ A) from a SLAC klystron (model 343). An assembly of focusing coils transports the electron beam into the drift tube. To insure good electron orbits, an aperture acting as an emittance selector is inserted which limits the electron beam radius to $r_b = 0.245$ cm so that only the inner portion of the beam is

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