Non-Maxwellian electron-energy distribution due to inelastic collisions in a z-pinch plasma

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Radiation from a hot plasma is produced by inelastic collisions of energetic electrons with highly ionized ions. In each collision these electrons lose a large amount of energy, which, in large part, is lost from the plasma as radiation and replenished by heating. Radiative cooling, principally in the tail of the electron distribution function, and heating, generally below thermal energies, modify the electron-energy distribution. This modification in turn affects the plasma properties. For a z-pinch neon plasma around 10^{19} cm^{-3} density and 200 eV temperature the radiative power decreases up to about 20%, although the radiation rate for an individual line could decrease by up to 100%.

I. INTRODUCTION

In moderate-atomic-number z-pinch plasmas, the radiation of interest generally consists of resonance lines and comes from inelastic collisions of electrons with highly stripped ions. This radiation depends on the excitation and ionization rates, i.e., integrals of the relevant collision cross sections with the electron-energy distribution function. In weakly ionized plasmas the electron-energy distribution may deviate strongly from a Maxwellian,[1] but a fully ionized plasma is much more collisional, and the distribution function is generally assumed to be Maxwellian. However, the electron-electron collision frequency decreases as the $\frac{3}{2}$ power of energy, while the inelastic-collision frequency decreases only as the $\frac{1}{2}$ power. Therefore electron-electron collisions are not necessarily so dominant for the energetic electrons that produce the resonance line radiation, and, consequently, inelastic collisions will reduce the energetic tail of the distribution function more than would be expected on the basis of the overall collisionality. This loss of energetic electrons reduces the amount of resonance line radiation produced when the electron temperature is much less than the excitation energy of the lines. Nonthermal electron distributions will result, therefore, from the radiation production process itself, even in highly ionized discharges, and these may need to be considered in making theoretical models of the process more accurate, and possibly in interpreting radiation measurements in terms of plasma properties. The purpose of this paper is to explore in some detail the effects of inelastic collisions on the equilibrium electron and ion distribution functions in a highly ionized neon plasma. The inelastic collisions to be considered are in the hydrogenlike, heliumlike, and lithiumlike ionization stages.

The equilibrium electron-energy distribution function will be calculated for plasma conditions typically observed in a neon gas puff z-pinch.[2,3] Table I contains these data. We will calculate nonthermal distributions for temperatures between 40 and 400 eV, and a density of 10^{19} neon ions/cm^3. In this parameter regime, the dominant ionization stage is eight-times-ionized He-like neon; also present are lithiumlike and hydrogenlike ions, together with their excited states.

Energy flow in these calculations is taken to be locally defined, i.e., the plasma is assumed to be effectively in coronal equilibrium and optically thin. In the electron calculation, it will thus be assumed that the ions deexcite only by emission of a photon. This is a good assumption when energy absorbed locally by the ions from the electrons in the inelastic excitation processes equals, in large part, the radiation loss. The plasma will be in equilibrium only when this local energy loss is compensated by an equal gain of energy from an external source. In z-pinch plasmas this heating is through compression, fast electron deposition, or Ohmic heating by an imposed electric field **E**; the latter heating mechanism is used in this paper. The electric field is then a parameter to be determined from the radiative power loss and the plasma conductivity. Sufficiently strong electric fields can cause electron runaway,[4] but the runaway issue is sidestepped here by the introduction into the analysis of a strong magnetic field **B** perpendicular to **E**. This is the situation away from the center of the z pinch.

One way to better understand the equilibrium behavior of electrons is to examine the rates of change of electron energy and momentum. The electron-energy distribution changes on these two main time scales. Momentum is randomized largely by electron-ion collisions. These are ($Z \sim 8$) more frequent than the electron-electron collisions, with collision frequency ν_{ee}, where the average charge state of the plasma Z is defined as n_e/n_i, where n_e and n_i are electron and ion densities, respectively. Therefore the electron distribution function will be isotropic for phenomena evolving on the electron-electron collision time scale $1/\nu_{ee}$. However, nonequilibrium changes in the energy distribution will occur on this time scale, because energy exchange is largely between electrons. Outside the runaway regime the electron-energy distribution function can be calculated by a first-order Legendre perturbation expansion around the isotropic distribution function, and this expansion is valid for all electron energies. The problem of runaway electrons arises in the presence of a strong electric field only along the pinch axis, where the magnetic field vanishes. This problem is deferred to future work.

Changes in the distribution function due to inelastic collisions should be related to the ratio between the inelastic and elastic collision frequencies. The inelastic collision cross section is typically expressed as

$$\sigma_{ab}(\varepsilon)=\pi a_0^2 \frac{\Omega_{ab}(\varepsilon/\varepsilon_{ab})}{g_a}\left[\frac{\mathcal{R}}{\varepsilon}\right], \tag{1}$$

where a_0 is the Bohr radius, $\varepsilon=mv^2/2$ is the electron kinetic energy, $\mathcal{R}=e^2/8\pi\epsilon_0 a_0$ is the Rydberg energy which is 13.6 eV, and Ω_{ab} is the collision strength for a transition from level a (with multiplicity g_a) to level b, with a threshold energy ε_{ab}. For K-shell modeling collision strengths can be found by scaling the hydrogenic[5] collision strengths Ω_{ab}^{H} and $\Omega_{ab}^{\mathrm{H,ex}}$ with the effective atomic number Z_{eff}, viz.,

$$\Omega_{ab}=(\Omega_{ab}^{\mathrm{H}}+\Omega_{ab}^{\mathrm{H,ex}})/Z_{\mathrm{eff}}^2 .$$

Data for the calculation of Ω_{ab}^{H} and $\Omega_{ab}^{\mathrm{H,ex}}$ are available in tables.[5,6]

The electron-electron collision cross section is (see the Appendix)

$$\sigma_{ee}(\varepsilon)=\pi a_0^2 8\ln\Lambda\left[\frac{\mathcal{R}}{\varepsilon}\right]^2, \tag{2}$$

where the Coulomb logarithm factor $8\ln\Lambda\sim 50$ reflects the increase over Rutherford scattering due to multiple small-angle deflections. Therefore the ratio between inelastic and elastic collision frequencies is

$$\frac{\nu_{ab}}{\nu_{ee}}=\frac{n_a\sigma_{ab}v}{n_e\sigma_{ee}v}=\frac{f_a(Z^2\Omega_{ab})}{8Zg_a\ln\Lambda}\left[\frac{\varepsilon}{Z^2\mathcal{R}}\right]. \tag{3}$$

Here $f_a=n_a/n_i<1$ is the ion fraction in the state a. Expression (3) becomes the inelastic-collision term in the equation for the electron distribution function when the electron energy ε is replaced with the thermal energy. Typically, Eq. (3) is small, a few percent for the plasma parameters of interest here. Moreover, it tends to scale as $1/Z$ as indicated by Eq. (3). Based on this ratio, one would expect a few percent change in the computed properties of the radiating plasma.

Another way to gauge the influence of inelastic collisions is to compare the power density emitted by the plasma, $P_{h\nu}$, to a power density in the exchange of energy between electrons, P_{ee}. Each photon that is emitted is created in an inelastic collision, which takes an energy $h\nu$ from the plasma. Electron-electron collisions correspond to an average energy exchange of the thermal energy kT. Therefore

$$\frac{P_{h\nu}}{P_{ee}}=\left[\frac{h\nu}{kT}\right]\left[\frac{\nu_{ab}}{\nu_{ee}}\right], \tag{4}$$

which differs by the factor $(h\nu/kT)\sim 3$ from the ratio between the collision times. On multiplying top and bottom by the number of electrons per unit length, one converts Eq. (4) into a ratio between powers per unit length. The radiative power per unit length in a neon z-pinch plasma[3] is perhaps 3 TW/cm. The power per unit length in electron-electron collisions is easily calculated. For the numbers in Table I, the power exchanged per electron-electron collision is $\sim$200 eV/ps, and, with 3×10^{18} electrons/cm, the power exchanged per unit length is $\sim$100 TW/cm. As in comparing the collision times, the comparison between these powers per unit length suggest a small but perhaps not negligible effect of the inelastic collisions, here on the order of 3%.

These estimates are lower than, but on the same order as, the computational results presented below. For our neon plasma, the electron distribution function is modified by the inelastic collisions sufficiently to decrease the radiative power up to 20% due to a decrease in the individual inelastic collision rates up to 100%. Therefore, a Maxwellian electron-energy distribution function generally remains a reasonable basis for estimating most plasma properties. However, for accurate work deviations from a Maxwellian must be taken into account.

II. MODEL EQUATIONS

Away from the axis, the isotropic part of the electron-energy distribution function for a radiating z-pinch plasma satisfies a Fokker-Planck equation[7–11] of the form discussed in the Appendix,

$$\sqrt{\epsilon}\frac{\partial f}{\partial\tau}=\frac{\partial}{\partial\epsilon}\left[Nf+D\frac{\partial f}{\partial\epsilon}+\mathcal{E}^2\zeta(\epsilon)\frac{\partial f}{\partial\epsilon}\right]-\alpha f+(\alpha f)_\Delta . \tag{5}$$

Here ϵ is the electron kinetic energy normalized to the plasma thermal energy, $\epsilon=\varepsilon/kT$, and τ is the time normalized to the electron-electron collision time, $\tau=\nu_{ee}t$.

The coefficient N is the incomplete normalization of the distribution,

$$N(\epsilon)=\int_0^\epsilon d\epsilon\,\epsilon^{1/2}f(\epsilon), \tag{6a}$$

TABLE I. Typical z-pinch plasma parameters.

	Symbol	Edge
Current	I	2 MA
Radius	r	1 mm
Magnetic field	B	1000 T
Gyrofrequency	$\omega_c=eB/m$	10^{14} s^{-1}
Gyroradius	r_e	10^{-5} cm
Plasma frequency	ω_p	10^{15} s^{-1}
Electron density	n_e	10^{20} cm^{-3}
Ion density	n_i	10^{19} cm^{-3}
Typical ionic charge	Z	10
Ionic size	a_0/Z	10^{-9} cm
Electron temperature	T_e	300 eV
Debye length	λ_D	10^{-6} cm
Plasma parameter	$n_e\lambda_D^3$	100
e-e collision time	τ_{ee}	1.5 ps
e-i	$\tau_{ei}=\tau_{ee}/Z$	0.15 ps
e-i energy exchange	$\tau_\epsilon=(M/m)\tau_{ei}$	
Collisionality	$\omega_{ce}\tau_{ei}$	$30\gg 1$

with $N(\infty)=1$. Similarly, D is

$$D(\epsilon)=\frac{2}{3}\left[\int_0^{\epsilon} d\epsilon\, \epsilon^{3/2} f(\epsilon)+\epsilon^{3/2}\int_{\epsilon}^{\infty} d\epsilon\, f\right] . \tag{6b}$$

For large energies, D approximates the average normalized energy and $D(\infty)=1$. In the absence of an electric field and inelastic collisions, the equilibrium solution to Eq. (5) is a Maxwellian distribution function, viz. for a Maxwellian $N=D$, and $f \propto \exp(-\int d\epsilon N/D)=\exp(-\epsilon)$.

In Eq. (5) the inelastic collisions are represented by the two terms with α, where

$$\alpha(\epsilon)=\frac{f_a \Omega_{ab}(\epsilon/\epsilon_{ab})}{8Zg_a \ln\Lambda}\left[\frac{kT}{\mathcal{R}}\right] \tag{7a}$$

depends on the collision strength $\Omega_{ab}(\epsilon/\epsilon_{ab})$ for a transition from the ground state a to an excited state b, with transition energy ϵ_{ab}, and on the fraction of ions f_a in the state a. The inelastic collision model for neon used in the Fokker-Planck equation is depicted in Fig. 1. The model contains the ten transitions shown connecting the ground state and three excited states of Li-like Ne VIII, with the ground states and two excited states of He-like Ne IX and H-like Ne X. All other transitions including deexcitations and ionizations from Li, He, and H excited states are ignored. This restricts the analysis to sufficiently low densities for which the coronal model approximation is valid, i.e., deexcitation of the excited states is dominated by radiative decay. The dominant energy loss from the plasma is to the He-like and the H-like resonance lines, indicated by H-$L\alpha$ and He-$L\alpha$, between the first excited states and the ground states. Figure 2 shows the associated collision strengths Ω_{ab} for Ne IX: $1\rightarrow 2$ indicates the collision strength from the ground state to the first excited state, etc. Ionization from the ground state is marked by $1\rightarrow\infty$. Collision strengths vanish for energies below the excitation thresholds, which for the $1\rightarrow 2$ transitions are equal to the energies of the resonance lines, 0.9 and 1.02 keV, respectively. Above threshold, $\Omega_{ab}\sim 0.05-0.2$ over an extended range in energy. At the left in Fig. 2 is the collision strength for ionization of Ne VIII. This collision strength is about ten times larger than the other collision strengths, and it has a smaller threshold, about 240 eV.

After an inelastic collision, the electron loses the energy of excitation of the ion, but the electron does not disappear. Hence the inelastic-collision term is balanced by a term that puts these electrons back at an energy $\epsilon'=\epsilon-\epsilon_i$, where i indexes a particular excitation process, $i=b\rightarrow a$. This is the generic term

$$(\alpha f)_{\Delta}=\alpha(\epsilon+\epsilon_i)f(\epsilon+\epsilon_i) . \tag{7b}$$

Because more than a single type of inelastic collision is considered, the inelastic-collision terms in Eq. (5) should be understood with an implicit summation,

$$-\alpha f+(\alpha f)_{\Delta}\Longrightarrow -\sum_i \alpha_i(\epsilon)f(\epsilon) + \sum_i \alpha_i(\epsilon+\epsilon_i)f(\epsilon+\epsilon_i) , \tag{7c}$$

although the summation sign has been suppressed. The summation is over the dominant excitation processes i shown in Fig. 1.

The ionization fractions f_a depend on the plasma temperature and density. The f_α's are obtained from equilibrium solutions to a set of rate equations[12] for the neon ion populations in Fig. 1. These equations contain the rate processes shown in Fig. 1 as incorporated into the Fokker-Planck equation. The rate coefficients are calculated by numerical integrations over the collision strength weighted by the electron energy-distribution function. Figure 3(a) shows how the ionization fractions

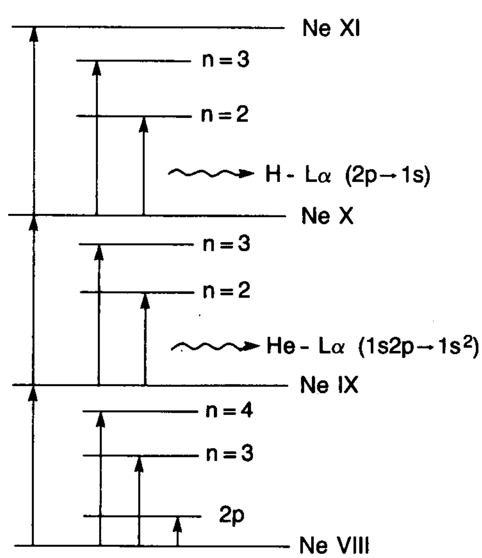

FIG. 1. The neon model, including the ground state and three excited states for Li-like Ne VIII, the ground state and two excited states for He-like Ne IX and for H-like Ne X. The resonance lines are indicated by H-$L\alpha$ and He-$L\alpha$.

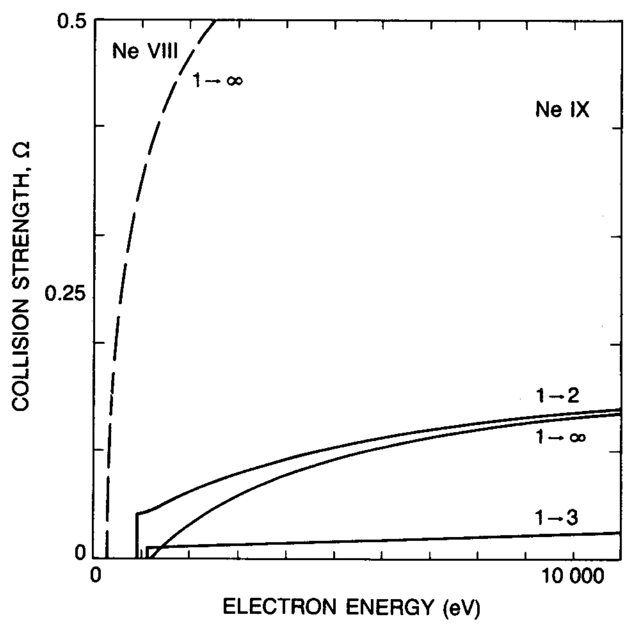

FIG. 2. Collision strength for heliumlike neon from the ground state ($n=1$) to the first ($n=2$) and the second ($n=3$) excited state. On the left is the collision strength for ionization of lithiumlike neon.

for the ground states depend on plasma temperature in the range of interest, 40–400 eV (calculated using Maxwellian averaged rate coefficients). At an ion density of 10^{19} ions/cm^3, the plasma consists almost entirely of heliumlike Ne IX ground states between 50 and 200 eV; at 300 eV the Ne X ground states are most abundant. Lithiumlike Ne VIII and the bare nucleus, Ne XI, almost never exceed 10% of the ions. Therefore the average ionization state Z in Fig. 3(b) has a value near 8. The ionization fractions for the excited states in Fig. 3(c) are likewise small. More detail in the ionization kinetics, i.e., the inclusion of more states and therefore more collision processes will not give large shifts to these populations at the low plasma densities for which these calculations are valid.

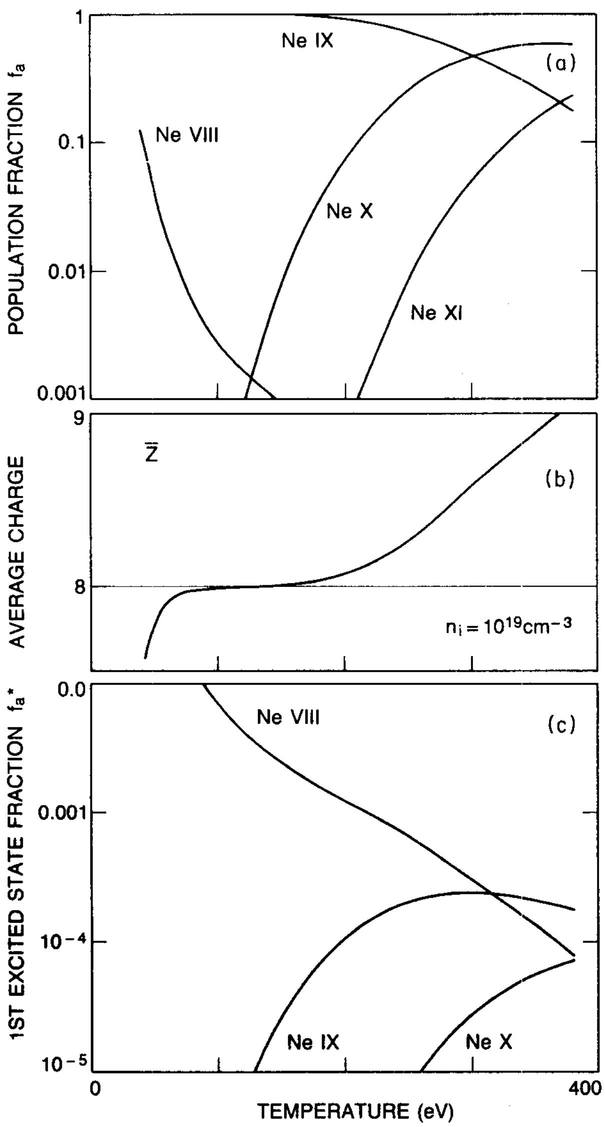

FIG. 3. (a) Ionization fractions of the Li-like, He-like, and H-like ionization stages vs temperature T. Over most of this temperature range the neon is almost exclusively He-like (Ne IX). (b) Average atomic number $\bar{Z}$ as a function of temperature for an optically thin neon plasma. (c) The fraction of ions in the excited states as marked. Few ions are excited, consistent with the neglect of excitation or ionization from these states.

Ohmic heating is represented by the electric field term, with

$$\zeta(\epsilon)=\left[\frac{4}{3(Z+1)}\right]\frac{\epsilon^3}{1+\Omega^2\epsilon^3/(Z+1)^2}\ , \tag{8}$$

where the normalized magnetic field is $\Omega=\omega_{ce}/\nu_{ee}$, and $\omega_{ce}=eB/m$ is the electron cyclotron frequency. The symbol Ω defines a normalized magnetic field, which should not be confused with the collision strength Ω_{ab} that enters the equation through α.

The normalized electric field $\mathcal{E}=E/E_0$, where E_0 is the Dreicer field that accelerates an electron from rest to the thermal energy in an electron-electron collision time, $eE_0/m\nu_{ee}=v_{\text{th}}$. An electric field on the order of E_0, i.e., $\mathcal{E}\sim 1$, would create runaway electrons in the absence of a magnetic field. However, in a z-pinch plasma, the magnetic field is sufficiently strong, $\Omega>1$, to prevent runaways off axis.

The Fokker-Planck equation (5) describes the evolution in time of the electron-energy distribution function $f(\epsilon,\tau)$, but interest here is limited to a determination of the stationary state $f(\epsilon)$. Stationarity is possible only when there is a balance between energy gains and losses, each of which is an average of the distribution function. Averages are defined by

$$\langle\phi(\epsilon)\rangle=\int d\epsilon\,\epsilon^{1/2}f(\epsilon)\phi(\epsilon)\ ,$$

and they change according to $d\langle\phi\rangle/d\tau=\int_0^\infty d\epsilon[\ ]$, where the [ ] stands for the right-hand side of Eq. (5).

In steady state, the plasma will not be ionizing or recombining and the number of electrons remains constant, $\int d\epsilon\sqrt{\epsilon}f(\epsilon)=1$. However, the excitation energy $\varepsilon_{ab}=\epsilon_i kT$ is lost in each inelastic collision, and restored by Ohmic heating. The average energy per electron $\langle\epsilon\rangle$ changes as

$$\frac{d\langle\epsilon\rangle}{d\tau}=\mathcal{E}^2\int_0^\infty d\epsilon\frac{\partial\zeta(\epsilon)}{\partial\epsilon}f(\epsilon)-\sum_i\epsilon_i X_i\ , \tag{9}$$

where X_i is the normalized rate constant for the inelastic collisions,

$$X_i=\int_{\epsilon_i}^\infty d\epsilon\,\alpha_i(\epsilon)f(\epsilon)\ . \tag{10}$$

In steady state, the energy content of the electron distribution is constant, $d\langle\epsilon\rangle/dt=0$. In this case the electric field and the plasma temperature are connected by

$$\mathcal{E}^2=\left[\sum_i\epsilon_i X_i\right]\Big/\left[\int_0^\infty d\epsilon\frac{\partial\zeta(\epsilon)}{\partial\epsilon}f(\epsilon)\right]\ . \tag{9'}$$

The stationary distribution function is now consistent with $\partial f/\partial t=0$ in Eq. (5).

Ohmic heating is strongest for those electrons with energies close to the peak of

$$\frac{\partial\zeta}{\partial\epsilon}=\left[\frac{4}{3(Z+1)}\right]\frac{3\epsilon^2}{[1+\Omega^2\epsilon^3/(Z+1)^2]^2}\ , \tag{11}$$

i.e., the electrons with energy ϵ around $(\frac{1}{2})^{1/3}[(Z+1)/\Omega]^{2/3}$. This is less than or around unity for our magnetic fields and, as a consequence, the energy is put into electrons at or below thermal. Energy must therefore be transported from $\epsilon \leq 1$ to the inelastic thresholds $\epsilon_i \gg 1$ and beyond.

We expect that the number of energetic electrons in the plasma described by Eq. (5) is reduced compared to a Maxwellian. Numerical results are contained in Sec. III. How much the inelastic collisions reduce the number of energetic electrons can be estimated analytically in simple cases, e.g., when the total collision strength $\sum \Omega_{ab}(\varepsilon/\varepsilon_{ab}) \propto \alpha(\epsilon)$ is constant, and the typical energy loss $\varepsilon_a = \sum \varepsilon_{ab}\Omega_{ab}/\sum \Omega_{ab}$ is much larger than the thermal energy, i.e., $\epsilon_a \gg 1$. Then the linearized Fokker-Planck equation becomes approximately, for large ϵ,

$$\frac{\partial}{\partial \epsilon}\left[f+\frac{\partial f}{\partial \epsilon}\right]=\alpha f ,$$

with constant $\alpha \ll 1$. The solution to this equation is $f \sim \exp[-\epsilon(1+\alpha)]$, which implies a reduction in the temperature of the distribution-function tail by the factor $1/(1+\alpha) \simeq 1-\alpha$.

A similar estimate is obtained for arbitrary collision strength provided that the energy loss is small compared to the temperature, i.e., $\epsilon_a \ll 1$. Then the equation becomes

$$\frac{\partial}{\partial \epsilon}\left[f+\frac{\partial f}{\partial \epsilon}\right]=\epsilon_a \frac{\partial \alpha(\epsilon) f}{\partial \epsilon} ,$$

which integrates to

$$f=\exp\left[-\int d\epsilon(1+\epsilon_a \alpha)\right] ,$$

which again points to a decrease in the temperature of the distribution-function tail, this time by $1-(\epsilon_a/\epsilon)\int d\epsilon\, \alpha(\epsilon)$.

A rate coefficient $X(T)$ whose integration over the collision strength is weighted only by the tail of the distribution function would reflect the decrease $-\Delta T$ in the tail temperature T. The relative decrease of the rate, $\Delta X/X$, becomes $\Delta X/X = -\Delta T\, \partial \ln(X)/\partial T$. Therefore, $\Delta X/X$ is more appreciable than $\Delta T/T$ because, typically, $\partial \ln(X)/\partial T \sim \varepsilon_a/kT^2$ and $\Delta X/X$ is larger than $(\Delta T/T)$ by a factor $\sim(\varepsilon_a/kT) \sim 4$ or so. These considerations do not apply for most transport coefficients, which are not wholly dependent on the distribution-function tail.

Note again that to get a stationary distribution function we have chosen Ohmic heating to balance the radiative power. However, in experiments with z-pinch plasmas, the energy can flow locally within the plasma through additional and perhaps more important channels, e.g., pdV work from plasma compression, or heat conduction if the plasma is nonuniform. The distribution of energy input over electron energies is then different from Ohmic heating, and the resulting equilibrium energy distribution will not necessarily be the same as computed here.

III. RESULTS

We have calculated the electron-energy distribution function under the influence of inelastic collisions by iterating twice on Eq. (5) (with $\partial f/\partial t = 0$). The first iterate is the solution to the linearized Fokker-Planck equation found by calculating $N(\epsilon)$ and $D(\epsilon)$ from a Maxwellian distribution; the second iterate, which provides a small correction, is then obtained from Eq. (5) with $N(\epsilon)$ and $D(\epsilon)$ calculated using the first iterate. The ground-state ionization fractions used in these calculations, which determine the strength of the inelastic contributions, are those from Fig. 3. Note that we have not yet addressed the problem of tailoring the iterations to guarantee convergence of this process. The second iterates shown in Figs. 4(a) and 4(b), however, should demonstrate the correct magnitude of the change in the distribution function caused by inelastic collisions.

The distribution function including inelastic collisions is compared in Fig.4 (a) to a Maxwellian distribution at the same temperature, a relatively low 54 eV. The

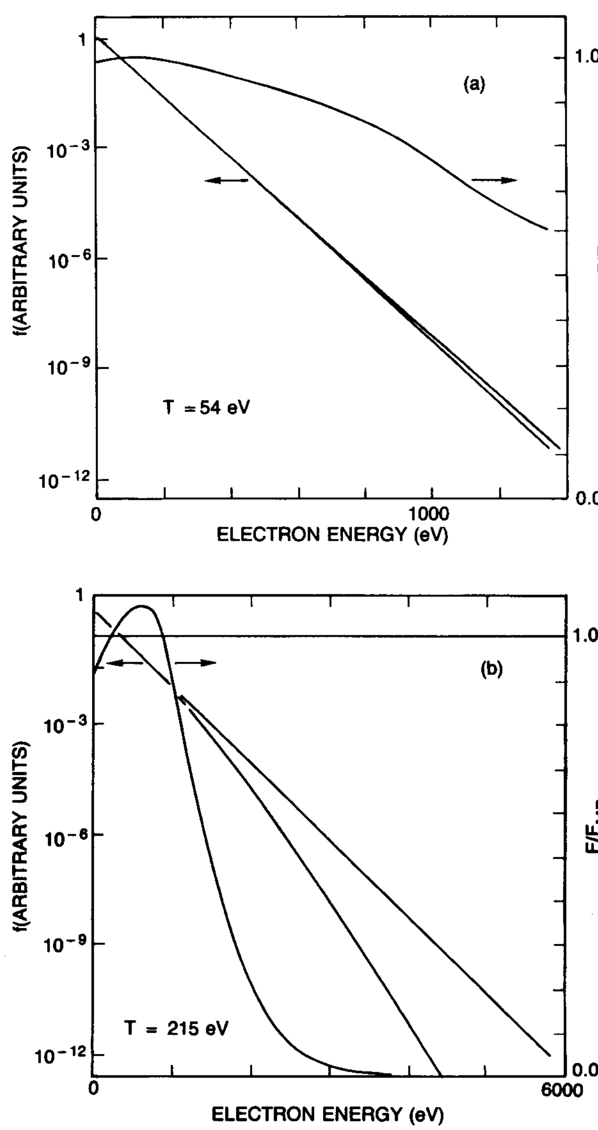

FIG. 4. (a) Logarithm of the isotropic electron distribution function $\ln[f(\epsilon)]$ for a neon plasma at $T=54$ eV (left scale), and the ratio $\gamma = f/f_{\rm MB}$ between this distribution function $f(\epsilon)$ and the Maxwellian $f_{\rm MB}(\epsilon)$ at the temperature (right scale). (b) Same for $T=215$ eV.

straight top line is the logarithm of the Maxwellian (left scale), covering over ten orders of magnitude. The perturbed distribution function is undiscernibly draped about this line until about ten times the thermal energy (lower line). To emphasize the deviations in f from a Maxwellian, Fig. 4 also contains the ratio $\gamma = f/f_{MB}$ between the two functions (right scale). The difference in f's, in this case, is insignificant for thermal energies, but, for five times thermal, the difference increases to about 5%. At a four-times–higher temperature of 215 eV, Fig. 4(b), the deviations from the Maxwellian are larger, especially towards the higher electron energies. However, for ε approximately five times thermal, the difference is still under about 10%. The distribution function is also depleted at electron energies much smaller than thermal. The reduction in the number of electrons at both extremes of the distribution function means that a surplus of electrons must show up between thermal and a few times thermal. The intermediate-energy region is filled by electrons that gain energy from the electric field on the low-energy side and electrons that have lost energy through inelastic collisions on the high-energy side.

The modified electron distribution function affects the excitation and ionization rates given by Eq. (10). These changes in rate coefficients, in turn, change the population distributions, which are shown in Fig. 5 as divisions of the ionization fractions of the various neon species that are calculated using non-Maxwellian rates by the same ionization fractions for the Maxwellian rates presented earlier in Fig. 3. Figure 5 demonstrates how the K-shell ionization states are progressively reduced in number as one goes up in energy; in the temperature range from 100 to 200 eV, the Ne X ground-state population is reduced by about 50% and Ne XI by more than threefold, as expected from a depleted high-energy electron population. These higher ionization stages are relatively rare in this temperature interval, below about 1%; the abundance of the main ionization state, the ground He-like Ne IX, remains relatively unchanged until, at temperatures higher than 200 eV, the greater abundance of Ne IX ground state relative to a Maxwellian can be seen as an $\sim 10\%$ increase. In contrast, the deviations caused by non-Maxwellian rates decrease with increasing temperature for Ne X and Ne XI ground states. In ongoing work we are investigating the fully self-consistent situation, including the effect of the changes in ionization fractions back into Eq. (5).

Excited states are also progressively less abundant when inelastic collisions deplete the number of high-energy electrons. Figure 6 shows reductions of up to 25% in the first excited state of Ne IX, and reductions of over 60% for the first excited state of Ne X. Since the decay of these states produces the overwhelming share of the plasma's kilovolt radiation, this radiation is proportionately reduced compared to its value when the electron distribution is taken to be a Maxwellian.

Ratios between the radiation output in the various lines can be used to estimate the plasma parameters. Figure 7 shows the line ratio between the hydrogenlike and heliumlike resonance line as a function of temperature for a Maxwellian, and for the distribution function, including inelastic collisions. The temperature estimate inferred from this line ratio changes by about 10% or 20 eV.

The sensitivity of plasma transport coefficients to deviations from a Maxwellian are also of interest. Because all electrons participate in transport to varying degrees, while excitation and ionization are possible only with electrons with energy above a threshold, it can be anticipated that transport coefficients are less sensitive to changes in the electron distribution than the rate

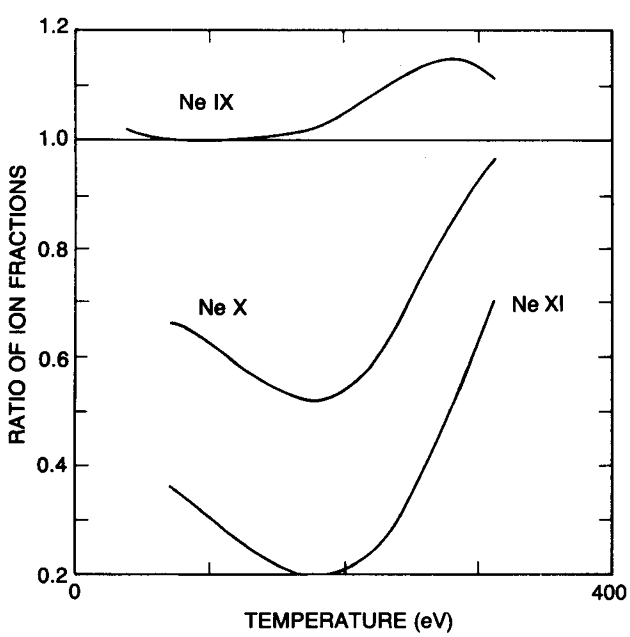

FIG. 5. The ionization fraction f^a in the specified ground state a with the self-consistent distribution function including inelastic collisions, relative to the ionization fraction f^a_{MB} for a Maxwellian distribution shown in Fig. 3(a).

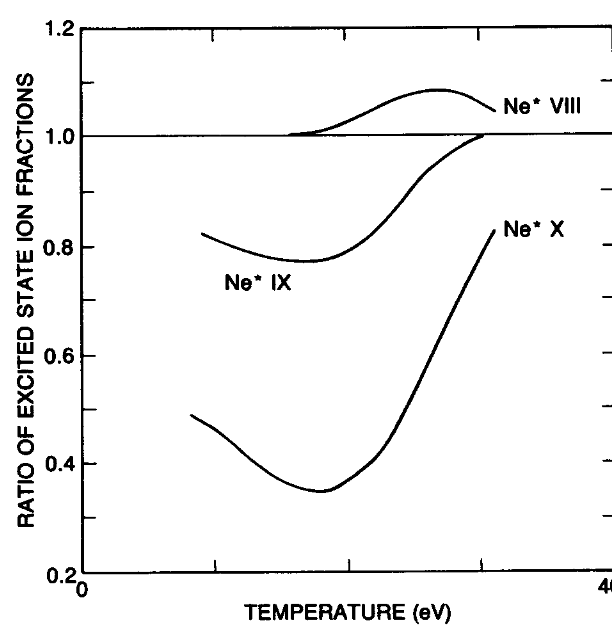

FIG. 6. The ionization fraction f^{a^*} of the excited state a^* with the self-consistent distribution function including inelastic collisions, relative to the ionization fraction $f^{a^*}_{MB}$ with a Maxwellian [compare Fig. 3(c)].

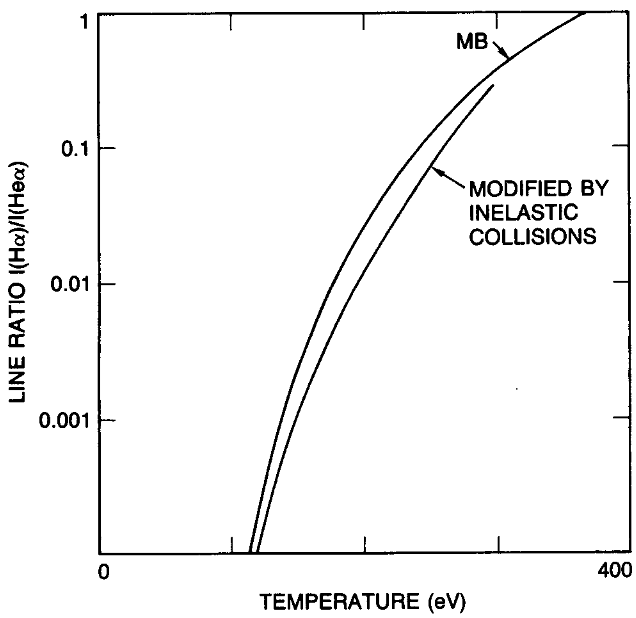

FIG. 7. The ratio between the He α and H α lines as a function of temperature T, and the change in this ratio when the distribution function changes through inelastic collisions.

coefficients are. As an example, the electrical conductivity deviates by less than 1% from the conductivity for a Maxwellian electron-energy distribution in a Lorentz gas.[13]

All the above results are for a typical magnetic field strength expected somewhere inside the z-pinch plasma, 300 T or 3 MG. Due to the strong peak around $\Omega^2\epsilon^3 \sim 1$ in $\partial\zeta/\partial\epsilon$, Eq. (8), the magnetic field influences the determination of where, in the electron distribution, Ohmic heating takes place, with corresponding quantitative but not qualitative changes in the various rates.

The sample plasma used here is heliumlike neon with some surrounding ionization stages, but the results obtained can be taken over to other heliumlike plasmas by scaling with atomic number Z. For an optically thin plasma in the absence of density effects, the same ionization abundances are obtained by increasing the plasma temperature proportional to the ionization energy, i.e., proportional to Z^2. The collision strength $\Omega = (Z^2\Omega)/Z^2$ scales roughly as $1/Z^2$; the factor $(Z^2\Omega)$ is the collision strength for a scaled isoelectronic ion, which is independent of Z to lowest order. Under these conditions, as noted earlier, the inelastic collision term,

$$\alpha \propto \frac{\Omega}{Z}\frac{T}{\mathcal{R}} ,$$

is inversely proportional to Z, because the decrease in the collision strength is compensated by the increase in the temperature. Therefore, the reductions in K-shell ionization that have been calculated here for neon are expected to be larger than those of higher atomic number plasmas.

IV. CONCLUSION

K-shell inelastic processes in an optically thin, coronal neon plasma change the electron-energy distribution function at a few times the thermal energy by up to $\sim$10%. These changes grow larger as one moves further out in the tail of the distribution function, up to an order of magnitude at approximately ten times the thermal energy. Consequently, the ion populations of the ground and excited states are reduced: when the thermal energy is comparable to the excitation energy, the reductions are minimal, but reductions up to 80% are calculated when the electron thermal energy is much less than the excitation energy. Transport coefficients that depend more on the bulk distribution of electrons are not significantly affected by K-shell inelastic collisions. Inelastic-collision–driven modifications should be more important in less ionized plasmas where a much larger number of inelastic-collision processes occur. Work along these lines is presently in progress.

ACKNOWLEDGMENTS

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APPENDIX: THE FOKKER-PLANCK EQUATION

The Fokker-Planck equation[7–11] is the appropriate equation for the electron distribution function $f(\mathbf{v})$ when multiple small-angle collisions dominate the electron dynamics. In a z-pinch plasma, where the average charge $\bar{Z}$ on the ions is large, e.g., $\bar{Z} \sim 10$, collisions between electrons and ions are much $(\bar{Z}\times)$ more frequent than electron-electron collisions. In electron-ion collisions the electron changes its direction without changing its energy, and the electron directions are already randomized well before the electrons are thermalized; thermalization is through collisions between electrons only. On the electron-electron time scale, the electron distribution function changes in time but remains almost isotropic, $f(\mathbf{v},t) \simeq f(v,t)$.

Relaxation toward equilibrium of $f(v,t)$ follows the equation

$$\frac{\partial f}{\partial t} = C(f,f) + C(E,B) + C^* , \qquad \text{(A1)}$$

where $C(f,f)$ is the Fokker-Planck collision term for multiple scattering between electrons, $C(E,B)$ represents the influence from an electric field E as modified by a magnetic field B, and C^* takes into account the inelastic processes. The electron-electron collision term is

$$C(f,f) = \frac{(\pi a_0^2 v_0 n) 8 \ln\Lambda v_0^3}{2v^2} \frac{\partial}{\partial v}\left[N(v)f(v) + D(v)\frac{\partial f}{\partial v}\right] . \qquad \text{(A2)}$$

The time scale factor multiplying the velocity derivatives is written in atomic units, because these are the natural units when atomic collisions are involved. The atomic units are the Bohr radius $a_0 = 5.292 \times 10^{-11}$ m, the Bohr cross section $\pi a_0^2 = 8.797 \times 10^{-21}$ m^2, and the Rydberg energy (in Joules) $\mathcal{R} = \epsilon^2/8\pi\epsilon_0 a_0 = 2.180$ aJ (1 aJ$=10^{-18}$ J), or in electron volts, $\mathcal{R} = e/8\pi\epsilon_0 a_0 = 13.606$ V. The atomic velocity $v_0 = 2.188 \times 10^6$ m/s is defined by

$\mathcal{R}=mv_0^2/2$. The Bohr cross section is the 90°-Rutherford scattering cross section at the atomic velocity v_0. Their product forms the normalization for the atomic rates, $k_0=\pi a_0^2 v_0=1.925\times10^{-14}$ m^3/s.

The normalized collision frequency ν_0 for a particular scattering s is obtained by multiplying with the density of the relevant scatter, $\nu_0^s=n^s k_0$; for electron-electron scattering, the scatterer density is the electron density n. The cumulative effect from many small-angle collisions between electrons that together give a 90° deflection is accounted for by the factor $8\ln\Lambda$, where $\Lambda\sim 9N_D$ is the Coulomb logarithm, and $N_D=4\pi n\lambda_D^3/3$ is the number of particles in a sphere with radius equal to the Debye length λ_D. The electron-electron collision frequency at v_0 is then $\nu_0=8\ln\Lambda\nu_0^e$.

The distribution function $f(v)$ is normalized to unity according to

$$N(v)=4\pi\int_0^v dv\, v^2 f(v),\quad N(\infty)=1\ . \tag{A2a}$$

The diffusion coefficient $D(v)$ is

$$D(v)=\frac{4\pi}{3v}\left[\int_0^v dv\, v^4 f(v)+v^3\int_v^\infty dv\, vf(v)\right]\ . \tag{A2b}$$

The electron temperature T is defined by averaging the electron energy $\varepsilon=mv^2/2$,

$$\tfrac{3}{2}kT=4\pi\int_0^\infty dv\, v^2\varepsilon f\ .$$

The thermal velocity v_{th} is defined by $mv_{\text{th}}^2/2=kT$. For velocities much larger than thermal, $D(v)$ becomes $D(v)=kT/mv=v_{\text{th}}^2/2v$. The Maxwell-Boltzmann distribution is

$$f^{\text{MB}}=(\pi v_{\text{th}}^2)^{-3/2}\exp(-v^2/v_{\text{th}}^2)\ .$$

The electric field term becomes

$$C(E,B)=(eE/3mv^2)\partial v^2 f_1/\partial v\ ,$$

where f_1 is the small anisotropy introduced in the distribution function $f(\mathbf{v})$ by the electric field. Expanding the electron distribution function as $f(\mathbf{v})=f_0(v)+f_1(v)\cos\theta+\cdots$, where θ is the angle between the electron velocity and electric field vectors. In equilibrium, f_1 is

$$f_1=\frac{eE}{m\nu_{ei}(1+\omega_{ce}^2/\nu_{ei}^2)}\frac{\partial f_0}{\partial v}\ ,$$

where $\nu_{ei}=(Z+1)\nu_{ee}$ is the electron-ion collision frequency. Equilibrium is assured because relaxation of f_1 toward equilibrium occurs on the electron-ion time scale $1/\nu_{ei}$, about Z times faster than the electron-electron collision time scale of Eq. (A2). As a corollary, electron-electron collisions and, *a fortiori*, inelastic collisions are negligible on the $1/\nu_{ei}$ time scale. The magnetic field B enters through the electron cyclotron frequency $\omega_{ce}=eB/m$. The electric field term is then

$$C(E,B)=\frac{1}{2v^2}\frac{\partial}{\partial v}\left[\frac{2e^2E^2v^2}{3m^2\nu_{ei}(1+\omega_{ce}^2/\nu_{ei}^2)}\frac{\partial f}{\partial v}\right]\ . \tag{A3}$$

The inelastic term C^* accounts for loss of electrons at large velocity when they lose energy on excitation or ionization of an ion, and their simultaneous reappearance at a lower energy. The inelastic cross section $\sigma_{ab}(\varepsilon)$ for transitions between state a and b is given by Eq. (1). Then C^* becomes

$$C^*=-n_a v\sigma_{ab}(\varepsilon)f(v)+(\ )_\Delta\ , \tag{A4}$$

where the first term represents the disappearance of electrons as compensated by the term $(\ )_\Delta$. The explicit for for this term is easiest when the variable is the electron energy rather than the velocity.

The natural velocity scale is the thermal velocity v_{th}, and the natural time scale is the electron-electron collisional time scale at v_{th}, $1/\nu$, where the collision frequency

$$\nu=\nu_0(v_{\text{th}}/v_0)^3(\ln\Lambda_0/\ln\Lambda)$$

or

$$\nu=\nu_0(kT/\mathcal{R})^{3/2}(\ln\Lambda_0/\ln\Lambda)\ .$$

Typically, $(\ln\Lambda_0/\ln\Lambda)\sim1$. Using $\xi=v/v_{\text{th}}$ and $\tau=t\nu$, Eq. (A1) becomes

$$\frac{\partial f(\xi,\tau)}{\partial\tau}=\frac{1}{2\xi^2}\frac{\partial}{\partial\xi}\left[N(\xi)f(\xi,\tau)+D(\xi)\frac{\partial f(\xi,\tau)}{\partial\xi}+\mathcal{E}^2\zeta(\xi)\frac{1}{2\xi}\frac{\partial f(\xi,\tau)}{\partial\xi}\right]-\frac{1}{\xi}[\alpha f(\xi,\tau)-(\ )_\Delta] \tag{A5}$$

where $N(\xi)$ and $D(\xi)$ are given by Eqs. (A2a) and (A2b) on replacing the velocity v by ξ. The second term is the Ohmic heating term, where the electric field $\mathcal{E}=E/E_0$ is normalized to a typical runaway electric field E_0 defined such that $eE_0/m\nu=v_{\text{th}}$. The function

$$\zeta(\xi)=4\xi^3(Z+1)^{-1}/[1+\Omega^2\xi^6/(Z+1)^2]$$

determines how the heating is divided over electron velocities; $\Omega=\omega_{ce}/\nu$ is the normalized magnetic field.

The last term αf is the inelastic loss term, where

$$\alpha=\frac{f_a\Omega_{ab}}{8Z\ln\Lambda g_a}\frac{kT}{\mathcal{R}}\ . \tag{A6}$$

In this term a convenient cancellation occurs between one velocity in $nv\sigma^*$ and another in the transformation $d\varepsilon=mv\,dv$ from velocity to energy, and the explicit energy dependence $(\mathcal{R}/\varepsilon)$ in the cross section $\sigma^*\propto\Omega(\varepsilon)(\mathcal{R}/\varepsilon)$. The gain term $(\ )_\Delta$ is best given directly in the normalized energy $\epsilon=\xi^2=\varepsilon/kT$. Then Eq. (A5) becomes Eq. (5) of the main text, with a Maxwellian $f(\epsilon)=(2/\sqrt{\pi})\exp(-\epsilon)$, and the coefficients $N(\epsilon)$ and $D(\epsilon)$ as given in Eq. (6).

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